1103
~~251~~ 3級パスノ空間∇_nノ中ノ
∇_{n-1}ニ就テ

朝長　康郎(東大學生)

§0. 3級パスノ理論ハ数年前ニ本部均先生ガ研究サレタ。[1] ソコデ今述ベタカラソノ超曲面ノ理論ヲ考ヘテ見ヨウ。

先ヅ、本部先生ニ隨ッテ、射影的媒変数ヲ持ツ様ナ3級パスノ空間∇_nノ構造ヲ決メル。基本的ナ量ノコトヲ簡單ニ説明スル。

(0.1) $T^{\lambda}=x^{(3)\lambda}+H^{\lambda}(x, x^{(1)}, x^{(2)})=0$

デ定義サレタ3級パスガ、射影的媒変数ヲ持ツト云フナラバH^{λ}ハ次ノ條件ヲ満足シナケレバナラヌ。

(0.2) a) $H^{\lambda}_{(1)\nu}x^{(1)\nu}+2H^{\lambda}_{(2)\nu}x^{(2)\nu}=3H^{\lambda}$

b) $H^{\lambda}_{(2)\nu}x^{(1)\nu}=-3x^{(2)\lambda}$

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(1) H. Hombu. Projektive Transformation eines Systems der gewöhnlichen Differential=gleichungen dritter Ordnung.
帝國学士院記事 Vol. 13.

H. Hombu. Die projektive Theorie der "Paths" 3-ter Ordnung $x^{(3)i}+H^{i}(x,x^{(1)},x^{(2)})=0$
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∇_n ノ基接續トシテ、次ノ二ツヲ採ル。

(0.3) a) $$\delta x^{(1)\lambda} = dx^{(1)\lambda} + \frac{1}{3} H^{\lambda}_{(2)\nu} dx^{\nu}$$

b) $$\delta x^{(2)\lambda} = dx^{(2)\lambda} + \frac{2}{3} H^{\lambda}_{(2)\nu} dx^{(1)\nu} + \frac{1}{3} H^{\lambda}_{(1)\nu} dx^{\nu}$$

普通ノ曲線ニ沿ッテハ $\dfrac{\delta x^{(1)\lambda}}{dt} = 0$，パスニ沿ッテハ $\dfrac{\delta x^{(2)\lambda}}{dt} = 0$ デアル。

∇_n ノヴェクトルノ共変微分ハ

(0.4) $$\delta v^{\lambda} = dv^{\lambda} + \omega^{\lambda}_{\mu} v^{\mu}.$$

茲ニ $$\omega^{\lambda}_{\mu} = \underset{(0)}{\overset{\varepsilon}{\Gamma}}{}^{\lambda}_{\mu\nu} dx^{\nu} + \underset{(1)}{\overset{\varepsilon}{\Gamma}}{}^{\lambda}_{\mu\nu} \delta x^{(1)\nu}$$

(0.5) a) $$\underset{(0)}{\overset{\varepsilon}{\Gamma}}{}^{\lambda}_{\mu\nu} = \frac{1}{3} H^{\lambda}_{(2)\mu(1)\nu} - \frac{2}{9} H^{\lambda}_{(2)\mu(2)\sigma} H^{\sigma}_{(2)\nu}$$

b) $$\underset{(1)}{\overset{\varepsilon}{\Gamma}}{}^{\lambda}_{\mu\nu} = \frac{2}{3} H^{\lambda}_{(2)\mu(2)\nu}$$

デ与ヘル。

(0.4) ハ又次ノ様ニモ書ケル。

$$\delta v^{\lambda} = \overset{(0)}{\nabla}_{\nu} v^{\lambda} \cdot dx^{\nu} + \overset{(1)}{\nabla}_{\nu} v^{\lambda} \cdot \delta x^{(1)\nu} + \overset{(2)}{\nabla}_{\nu} v^{\lambda} \delta x^{(2)\nu}$$

茲ニ

(0.6) $$\overset{(0)}{\nabla}_{\nu} v^{\lambda} = \overset{(0)}{\overline{\nabla}}_{\nu} v^{\lambda} + \underset{(0)}{\overset{\varepsilon}{\Gamma}}{}^{\lambda}_{\mu\nu} v^{\mu}, \quad \overset{(1)}{\nabla}_{\nu} v^{\lambda} = \overset{(1)}{\overline{\nabla}}_{\nu} v^{\lambda} + \underset{(0)}{\overset{\varepsilon}{\Gamma}}{}^{\lambda}_{\mu\nu} v^{\mu}$$

$$\overset{(2)}{\nabla}_{\nu} v^{\lambda} = \overset{(2)}{\overline{\nabla}}_{\nu} v^{\lambda}$$

(0.7) a) $$\overline{\nabla}^{(0)}_{\nu} = \frac{\partial}{\partial x^{\nu}} - \frac{1}{3} H^{\mu}_{(2)\nu} \frac{\partial}{\partial x^{(1)\mu}} - \left(\frac{1}{3} H^{\mu}_{(1)\nu} - \frac{2}{9} H^{\mu}_{(2)\sigma} H^{\sigma}_{(2)\nu} \right) \frac{\partial}{\partial x^{(2)\mu}}$$

b) $$\overline{\nabla}^{(1)}_{\nu} = \frac{\partial}{\partial x^{(1)\nu}} - \frac{2}{3} H^{\mu}_{(2)\nu} \frac{\partial}{\partial x^{(2)\mu}}$$

c) $$\overline{\nabla}^{(2)}_{\nu} = \frac{\partial}{\partial x^{(2)\nu}}$$

∇_κ ノ曲率テンソル[1]ハ v^λ ヲ任意ノヴェクトルトスレバ

(0.8) $$\underset{2}{\delta}\underset{1}{\delta} v^{\lambda} - \underset{1}{\delta}\underset{2}{\delta} v^{\lambda} = v^{\mu}\Big[\big(R^{\lambda(0)(0)}_{\mu\sigma\pi} + \overset{*}{\Gamma}{}^{\lambda}_{(1)\mu\tau} S^{(0)(0)\tau}_{\sigma\pi(1)} \big) \underset{1}{d}x^{\sigma} \underset{2}{d}x^{\pi} + R^{\lambda(1)(1)}_{\mu\sigma\pi} \underset{1}{\delta}x^{(1)\sigma} \underset{2}{\delta}x^{(1)\pi} + R^{\lambda(0)(1)}_{\mu\sigma\pi} \big[\underset{1}{d}x^{\sigma} \underset{2}{\delta}x^{(1)\pi} \big] + R^{\lambda(0)(2)}_{\mu\sigma\pi} \big[\underset{1}{d}x^{\sigma} \underset{2}{\delta}x^{(2)\pi} \big] + R^{\lambda(1)(2)}_{\mu\sigma\pi} \big[\underset{1}{\delta}x^{(1)\sigma} \underset{2}{\delta}x^{(2)\pi} \big] \Big]$$

ノ如キ意味ヲ有シ、ソノ形ハ次ノ通リデアル。

(0.9) a) $$R^{\lambda(0)(0)}_{\mu\sigma\pi} = \overline{\nabla}^{(0)}_{\pi} \underset{(0)}{\check{\Gamma}}{}^{\lambda}_{\mu\sigma} - \overline{\nabla}^{(0)}_{\sigma} \underset{(0)}{\check{\Gamma}}{}^{\lambda}_{\mu\pi} + \underset{(0)}{\check{\Gamma}}{}^{\tau}_{\mu\sigma} \underset{(0)}{\check{\Gamma}}{}^{\lambda}_{\tau\pi} - \underset{(0)}{\check{\Gamma}}{}^{\tau}_{\mu\pi} \underset{(0)}{\check{\Gamma}}{}^{\lambda}_{\tau\sigma}$$

b) $$R^{\lambda(1)(1)}_{\mu\sigma\pi} = \overline{\nabla}^{(1)}_{\pi} \underset{(1)}{\check{\Gamma}}{}^{\lambda}_{\mu\sigma} - \overline{\nabla}^{(1)}_{\sigma} \underset{(1)}{\check{\Gamma}}{}^{\lambda}_{\mu\pi} + \underset{(1)}{\check{\Gamma}}{}^{\tau}_{\mu\sigma} \underset{(1)}{\check{\Gamma}}{}^{\lambda}_{\tau\pi} - \underset{(1)}{\check{\Gamma}}{}^{\tau}_{\mu\pi} \underset{(1)}{\check{\Gamma}}{}^{\lambda}_{\tau\sigma}$$

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(1) 曲率テンソルハ全然筆者ガ附加ヘタモノデアル。

c) $$R^{\lambda(0)(1)}_{\mu\sigma\pi} = \overline{\nabla}^{(1)}_{\pi}\underset{(0)}{\Gamma}{}^{\nu\lambda}_{\mu\sigma} - \overline{\nabla}^{(0)}_{\sigma}\underset{(1)}{\Gamma}{}^{\nu\lambda}_{\mu\pi} - \underset{(0)}{\Gamma}{}^{\nu\lambda}_{\varepsilon\sigma}\underset{(1)}{\Gamma}{}^{\nu\varepsilon}_{\mu\pi}$$

$$+\underset{(1)}{\Gamma}{}^{\nu\lambda}_{\varepsilon\pi}\underset{(0)}{\Gamma}{}^{\nu\varepsilon}_{\mu\sigma} + \underset{(1)}{\Gamma}{}^{\nu\lambda}_{\mu\varepsilon}\underset{(0)}{\Gamma}{}^{\nu\varepsilon}_{\sigma\pi}$$

d) $$R^{\lambda(0)(2)}_{\mu\sigma\pi} = \overline{\nabla}^{(2)}_{\pi}\underset{(0)}{\Gamma}{}^{\nu\lambda}_{\mu\sigma} + \frac{1}{3}\underset{(1)}{\Gamma}{}^{\nu\lambda}_{\mu\tau}\overline{\nabla}^{(2)}_{\pi}H^{\tau}_{(2)\sigma}$$

e) $$R^{\lambda(1)(2)}_{\mu\sigma\pi} = \overline{\nabla}^{(2)}_{\pi}\underset{(1)}{\Gamma}{}^{\nu\lambda}_{\mu\sigma}$$

又捩率テンソル[2]ハ次ノヤウナ意味ヲ持ツ。

(0.10)

a) $$\underset{2}{\delta}\underset{1}{d}x^{\lambda} - \underset{1}{\delta}\underset{2}{d}x^{\lambda} = S^{(0)(0)\lambda}_{\mu\sigma(0)}\underset{1}{d}x^{\mu}\underset{2}{d}x^{\sigma}$$

$$+S^{(0)(1)\lambda}_{\mu\sigma(0)}\left[\underset{1}{d}x^{\mu}\underset{2}{\delta}x^{(1)\sigma}\right]$$

b) $$\underset{2}{\delta}\underset{1}{\delta}x^{(1)\lambda} - \underset{1}{\delta}\underset{2}{\delta}x^{(1)\lambda} = S^{(0)(0)\lambda}_{\mu\sigma(1)}\underset{1}{d}x^{\mu}\underset{2}{d}x^{\sigma}$$

$$+S^{(0)(2)\lambda}_{\mu\sigma(1)}\left[\underset{1}{d}x^{\mu}\underset{2}{\delta}x^{(2)\sigma}\right] + S^{(0)(1)\lambda}_{\mu\sigma(1)}\left[\underset{1}{d}x^{\mu}\underset{2}{\delta}x^{(1)\sigma}\right]$$

c) $$\underset{2}{\delta}\underset{1}{\delta}x^{(2)\lambda} - \underset{1}{\delta}\underset{2}{\delta}x^{(2)\lambda} = S^{(0)(0)\lambda}_{\mu\sigma(2)}\underset{1}{d}x^{\mu}\underset{2}{d}x^{\sigma}$$

$$+S^{(1)(1)\lambda}_{\mu\sigma(2)}\underset{1}{\delta}x^{(1)\mu}\underset{2}{\delta}x^{(1)\sigma} + S^{(0)(1)\lambda}_{\mu\sigma(2)}\left[\underset{1}{d}x^{\mu}\underset{2}{\delta}x^{(1)\sigma}\right]$$

a) $$S^{(0)(0)\lambda}_{\mu\sigma(0)} = \underset{(0)}{\Gamma}{}^{\nu\lambda}_{\mu\sigma} - \underset{(0)}{\Gamma}{}^{\nu\lambda}_{\sigma\mu}$$

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(2) 捩率テンソルハ会都、本部先生ノアタヘラレタモノノ２倍トシテアル。

b) $S^{(0)(1)\lambda}_{\mu\sigma(0)} = \underset{(1)}{\overset{k}{\Gamma}}{}^{\lambda}_{\mu\sigma}$

c) $S^{(0)(0)\lambda}_{\mu\sigma(1)} = \frac{1}{3}\overline{\nabla}^{(0)}_{\sigma} H^{\lambda}_{(2)\mu} - \frac{1}{3}\overline{\nabla}^{(0)}_{\mu} H^{\lambda}_{(2)\sigma}$

d) $S^{(0)(2)\lambda}_{\mu\sigma(1)} = \frac{1}{2}\underset{(1)}{\overset{k}{\Gamma}}{}^{\lambda}_{\mu\sigma}$

e) $$S^{(0)(0)\lambda}_{\mu\sigma(2)} = \frac{1}{3}\overline{\nabla}^{(0)}_{\sigma} H^{\lambda}_{(1)\mu} - \frac{2}{9}\left(\overline{\nabla}^{(0)}_{\mu} H^{\lambda}_{(2)\pi}\right)\cdot H^{\pi}_{(2)\sigma}$$
$$- \frac{1}{3}\overline{\nabla}^{(0)}_{\mu} H^{\lambda}_{(1)\sigma} + \frac{2}{9}\left(\overline{\nabla}^{(2)}_{\sigma} H^{\lambda}_{(2)\pi}\right) H^{\pi}_{(2)\mu}$$

f) $S^{(1)(1)\lambda}_{\mu\sigma(2)} = S^{(0)(0)\lambda}_{\mu\sigma(0)}$

g) $S^{(0)(1)\lambda}_{\mu\sigma(2)} = \frac{1}{3}\overline{\nabla}^{(1)}_{\sigma} H^{\lambda}_{(1)\mu} - \frac{2}{3}\overline{\nabla}^{(0)}_{\mu} H^{\lambda}_{(2)\sigma} - \frac{2}{9} H^{c}_{(2)\mu}\overline{\nabla}^{(1)}_{\sigma} H^{\lambda}_{(2)c}$

h) $S^{(0)(1)\lambda}_{\mu\sigma(1)} = S^{(0)(0)\lambda}_{\mu\sigma(0)}$

§1. V_n ノ中ノ超曲面 V_{n-1} ノ方程式ヲ

$$x^{\lambda} = x^{\lambda}(y, y^{\dot{1}}, y^{\dot{2}}, \ldots\ldots, y^{n-1})$$

トスルトキ、此ノ V_{n-1} ノ上ノ與ヘラレタ3級パス(矢張リ射影的媒変數ヲ持ツ)

$$T^{i} = y^{(3)i} + H^{i}(y, y^{(1)}, y^{(2)}) = 0$$

$$(i = \dot{1}, \dot{2}, \ldots\ldots, n-1)$$

ノ H^{i} カラ全ク V_n ノトキト仝様ニ諸種ノ量ヲ決メル。

但シ，ギリシヤ字ハ V_n ニ關スル index ニ，ラテン字ハ V_{n-1} ニ關スル index ヲ用ヒルコトヽスル。今

$$(1.1)\quad \mathfrak{J}^{\lambda}=H^{\lambda}-\xi^{\lambda}_{i}H^{i}+3\xi^{\lambda}_{jk}y^{(2)j}y^{(1)k}+\xi^{\lambda}_{jkh}y^{(1)j}y^{(1)k}y^{(1)h}$$

ト置クト

$$\left(\text{茲ニ}\ \xi^{\lambda}_{i}=\frac{\partial x^{\lambda}}{\partial y^{i}},\ \xi^{\lambda}_{jk}=\frac{\partial^{2}x^{\lambda}}{\partial y^{j}\partial y^{k}},\ \xi^{\lambda}_{jkh}=\frac{\partial^{3}x^{\lambda}}{\partial y^{j}\partial y^{k}\partial y^{h}}\right)$$

$\mathfrak{J}^{\lambda}$ ハ V_n ノヴエクトルデ V_{n-1} ノ scalar トナル。

假定ニヨリ

$$(1.2)\quad a)\ \mathfrak{J}^{\lambda}_{(1)s}y^{(1)s}+2\mathfrak{J}^{\lambda}_{(2)s}y^{(2)s}=3\mathfrak{J}^{\lambda}$$

$$b)\ \mathfrak{J}^{\lambda}_{(2)s}y^{(1)s}=0$$

デナケレバナラナイ，(1.1)カラ得ラレル。

$$(1.3)\quad a)\ \mathfrak{J}^{\lambda}_{(1)s}=H^{\lambda}_{(1)\nu}\xi^{\nu}_{is}+2H^{\lambda}_{(2)\nu}\xi^{\nu}_{is}y^{(1)i}-\xi^{\lambda}_{i}H^{i}_{(1)s}+3\xi^{\lambda}_{is}y^{(2)i}+3\xi^{\lambda}_{sjk}y^{(1)j}y^{(1)k}$$

$$b)\ \mathfrak{J}^{\lambda}_{(2)s}=H^{\lambda}_{(2)\nu}\xi^{\nu}_{is}-\xi^{\lambda}_{i}H^{i}_{(2)s}+3\xi^{\lambda}_{is}y^{(1)i}$$

ヲ利用スレバ V_n ノ量亜ニ對シテ

(1.4) a) $\overline{\nabla}^{0}_{l}\Phi = \xi^{\lambda}_{l}\overline{\nabla}^{(0)}_{\lambda}\Phi + \frac{1}{3}J^{\lambda}_{(2)l}\overline{\nabla}^{(1)}_{\lambda}\Phi$

$$+\frac{1}{3}\left(\overline{\nabla}^{(1)}_{l}J^{\lambda}\right)\overline{\nabla}^{(2)}_{\lambda}\Phi$$

b) $\overline{\nabla}^{(1)}_{l}\Phi = \xi^{\lambda}_{l}\overline{\nabla}^{(1)}_{\lambda}\Phi + \frac{2}{3}J^{\lambda}_{(2)l}\overline{\nabla}^{(2)}_{\lambda}\Phi$

c) $\overline{\nabla}^{(2)}_{l}\Phi = \xi^{\lambda}_{l}\overline{\nabla}^{(2)}_{\lambda}\Phi$

ノ如キ關係ガ成ル。

又 ∇_{n-1} ニ属スル $x^{(1)\lambda}$, $x^{(2)\lambda}$ 即チ

$$x^{(1)\lambda} = \xi^{\lambda}_{i}y^{(1)i}, \quad x^{(2)\lambda} = \xi^{\lambda}_{i}y^{(2)i} + \xi^{\lambda}_{ij}y^{(1)i}y^{(1)j}$$

ヲ用ヒタトキハ

(1.5) a) $\delta x^{(1)\lambda} = \xi^{\lambda}_{i}\delta y^{(1)i} + \frac{1}{3}J^{\lambda}_{(2)i}dy^{i}$

b) $\delta x^{(2)\lambda} = \xi^{\lambda}_{i}\delta y^{(2)i} + \frac{2}{3}J^{\lambda}_{(2)i}\delta y^{(1)i} + \frac{1}{3}\left(\overline{\nabla}^{(1)}_{j}J^{\lambda}\right)dy^{j}$

ガ成立スルカラ、$\delta\xi^{\lambda}_{j}$ ヲ計算スレバ

(1.6) $\delta\xi^{\lambda}_{j} = F^{\lambda}_{jk}dy^{k} + \frac{2}{3}J^{\lambda}_{(2)j(2)k}\delta y^{(1)k}$

(1.7) $F^{\lambda}_{jk} = \xi^{\lambda}_{jk} + \underset{(0)}{\overset{k}{\Gamma}}{}^{\lambda}_{\mu\nu}\xi^{\mu}_{j}\xi^{\nu}_{\cdot k} - \xi^{\lambda}_{i}\underset{(0)}{\overset{k}{\Gamma}}{}^{i}_{jk}$

$$+\frac{1}{3}\underset{(1)}{\overset{k}{\Gamma}}{}^{\lambda}_{\mu\nu}\xi^{\nu}_{\cdot j}J^{\nu}_{(2)k}$$

(1.8) $\frac{2}{3}J^{\lambda}_{(2)j(2)k} = \underset{(1)}{\overset{k}{\Gamma}}{}^{\lambda}_{\mu\nu}\xi^{\mu}_{j}\xi^{\nu}_{,k} - \xi^{\lambda}_{i}\underset{(1)}{\overset{k}{\Gamma}}{}^{i}_{jk}$

ノ如キ関係ガ生ズル。

(1.3) ト (1.4) カラ

$$(1.9)\quad \Gamma^{\lambda}_{jk} = \frac{1}{3}\overset{(1)}{\nabla}_k \mathcal{T}^{\lambda}_{(2)j}$$

トナルコトガ直グ分ル。(1.6) ヨリ

V_{n-1} ニ切スルヴェクトル $v^{\lambda} = \xi^{\lambda}_i v^i$ ヲ V_{n-1} ニ沿ッテ共変微分スレバ

$$(1.10)\quad \delta v^{\lambda} = \xi^{\lambda}_i \delta v^i + v^j \Big(\Gamma^{\lambda}_{jk} dy^k + \frac{2}{3}\mathcal{T}^{\lambda}_{(2)j(2)k}\delta y^{(1)k}\Big)$$

コヽデ、到ル所 $\mathcal{T}^{\lambda}=0$ ノ如キ V_{n-1} ヲ *totally geodesic* V_{n-1} ト名付ケルナラバ (1.1) ヲ書キ直シテ得ラレル。

$$\mathcal{T}^{\lambda} = T^{\lambda} - \xi^{\lambda}_i T^i$$

ヨリ, *totally geodesic* V_{n-1} デハ、V_{n-1} ノパスハ必ズ V_n ノパストナッテキルコトガ分ル。次ニ到ル所 $\mathcal{T}^{\lambda}_{(2)k} = 0$ ノ如キ V_{n-1} ヲ *semi-geodesic* V_{n-1} ト呼ブコトニスレバ、(1.9) (1.2) ヨリ　コノ V_{n-1} ハ次ノ性質ヲ有ツ。

a) $\delta\xi^{\lambda}_j = 0$

b) $\mathcal{T}^{\lambda}$ ガ $x^{(1)i}$ ノ三次ノ斉次式

即チ、*semi-geodesic* V_{n-1} ハ、ソレニ切スルヴェクトル v^{λ} ヲ V_n ノ意味デ V_{n-1} ニ沿ッテ平行ニ移動スレバ

$\underline{\nabla_{n-1}}$ デモ平行ニ移動スル。ト云フ特徴ヲ持ツ。モトニ帰ッテ

(1.7) (1.8) ヨリ直チニ

$$(1.11)\quad \xi^{\mu}_{j} w^{\lambda}_{\mu} = \xi^{\lambda}_{i} w^{i}_{j} - \xi^{\lambda}_{jk} dy^{k} + F^{\lambda}_{jk} dy^{k} + \frac{2}{3} J^{\lambda}_{(2)j(2)k}\, \delta y^{(1)k}$$

ヲ得ル。

§2. (1.11) ノ積分可能条件ヲ求メルニハ (1.6)(1.10) ヲ利用シテ

$$v^{\lambda} = \xi^{\lambda}_{j} v^{j}$$

ナル任意ノ v^{λ} ニ對シテ $\delta_2\delta_1 v^{\lambda} - \delta_1\delta_2 v^{\lambda}$ ヲ計算シタ方ガヨイ。即チ出テ来ル條件ハ次ノ如シ。

(2,1)

$$\begin{aligned}
\text{a)}\quad & \overset{\vee}{R}{}^{\lambda(0)(0)}_{\mu\sigma\pi}\, \xi^{\mu}_{.j} \xi^{\sigma}_{.k} \xi^{\pi}_{h} + \frac{1}{9} R^{\lambda(1)(1)}_{\mu\sigma\pi} J^{\sigma}_{(2)k} J^{\pi}_{(2)h} \xi^{\mu}_{j} \\
& \qquad + \frac{1}{3} R^{\lambda(0)(1)}_{\mu\sigma\pi} \xi^{\mu}_{j} \left(\xi^{\sigma}_{.k} J^{\pi}_{(2)h}\, \xi^{\sigma}_{.h} J^{\pi}_{(2)k} \right) \\
& + \frac{1}{3} R^{\lambda(0)(2)}_{\mu\sigma\pi} \xi^{\mu}_{j} \left[\xi^{\sigma}_{k} \left(\overset{(1)}{\overline{\nabla}}_{h} J^{\pi} \right) - \xi^{\sigma}_{h} \left(\overset{(1)}{\overline{\nabla}}_{k} J^{\pi} \right) \right] \\
& + \frac{1}{9} R^{\lambda(1)(2)}_{\mu\sigma\pi} \xi^{\mu}_{j} \left[J^{\sigma}_{(2)k} \left(\overset{(1)}{\overline{\nabla}}_{h} J^{\pi} \right) - J^{\sigma}_{(2)h} \left(\overset{(1)}{\overline{\nabla}}_{k} J^{\pi} \right) \right] \\
& = \xi^{\lambda}_{i} \overset{\vee}{R}{}^{i(0)(0)}_{jkh} + \overset{(0)}{\nabla}_{h} F^{\lambda}_{jk} - \overset{(0)}{\nabla}_{k} F^{\lambda}_{jh} \quad {}^{(1)} \\
& + F^{\lambda}_{ja} S^{(0)(0)a}_{kh(0)} + \frac{2}{3} J^{\lambda}_{(2)j(2)a} S^{(0)(0)a}_{kh(1)}
\end{aligned}$$

脚註(1) 次頁へ

茲ニ $\overset{\times}{R}{}^{\lambda(0)(0)}_{\mu\sigma\pi} = R^{\lambda(0)(0)}_{\mu\sigma\pi} + \underset{(1)}{\Gamma}{}^{\times\lambda}_{\mu\tau} S^{(0)(0)\tau}_{\sigma\pi(1)}$

b) $R^{\lambda(1)(1)}_{\mu\sigma\pi}\xi^{\mu}_{,j}\xi^{\sigma}_{k}\xi^{\pi}_{h}$

$+\frac{2}{3}\left(R^{\lambda(1)(2)}_{\mu\sigma\pi}\xi^{\sigma}_{k}\mathfrak{J}^{\pi}_{(2)h} - R^{\lambda(1)(2)}_{\mu\sigma\pi}\xi^{\sigma}_{h}\mathfrak{J}^{\pi}_{(2)k}\right)\xi^{\mu}_{,j}$

$=\xi^{\lambda}_{i}R^{i(1)(1)}_{jkh} + \frac{2}{3}\left(\overset{(1)}{\nabla}_{h}\mathfrak{J}^{\lambda}_{(2)j(2)k} - \overset{(1)}{\nabla}_{k}\mathfrak{J}^{\lambda}_{(2)j(2)h}\right)$

c) $R^{\lambda(0)(1)}_{\mu\sigma\pi}\xi^{\mu}_{j}\xi^{\sigma}_{k}\xi^{\pi}_{h} + \frac{1}{3}R^{\lambda(1)(1)}_{\mu\sigma\pi}\xi^{\mu}_{j}\mathfrak{J}^{\sigma}_{(2)k}\xi^{\pi}_{h}$

$+\frac{2}{3}R^{\lambda(0)(2)}_{\mu\sigma\pi}\xi^{\mu}_{,j}\xi^{\sigma}_{k}\mathfrak{J}^{\pi}_{(2)h}$

$-\frac{1}{3}R^{\lambda(1)(2)}_{\mu\sigma\pi}\xi^{\mu}_{j}\xi^{\sigma}_{h}\left(\overset{(1)}{\overline{\nabla}}_{k}\mathfrak{J}^{\pi}_{\cdot}\right)$

$+\frac{2}{9}R^{\lambda(1)(2)}_{\mu\sigma\pi}\mathfrak{J}^{\sigma}_{(1)k}\mathfrak{J}^{\pi}_{(2)h}\xi^{\mu}_{j}$

$=\xi^{\lambda}_{i}R^{i(0)(1)}_{jkh} + \overset{(1)}{\nabla}_{h}F^{\lambda}_{jk} - \frac{2}{3}\overset{(0)}{\nabla}_{k}\mathfrak{J}^{\lambda}_{(2)j(2)h}$

$+F^{\lambda}_{ja}S^{(0)(1)a}_{kh(0)} + \frac{2}{3}\mathfrak{J}^{\lambda}_{(2)j(2)a}S^{(0)(1)a}_{kh(1)}$

d) $R^{\lambda(0)(2)}_{\mu\sigma\pi}\xi^{\mu}_{,j}\xi^{\sigma}_{k}\xi^{\pi}_{h} + \frac{1}{3}R^{\lambda(1)(2)}_{\mu\sigma\pi}\xi^{\mu}_{j}\mathfrak{J}^{\sigma}_{(2)k}\xi^{\pi}_{h}$

$=\xi^{\lambda}_{i}R^{i(0)(2)}_{jkh} + \overset{(2)}{\nabla}_{h}F^{\lambda}_{jk}$

$+\frac{2}{3}\mathfrak{J}^{\lambda}_{(2)j(2)a}S^{(0)(2)a}_{kh(1)}$

著者脚註

$dF^{\lambda}_{jk} + \omega^{\lambda}_{\mu}F^{\mu}_{jk} - \omega^{a}_{j}F^{\lambda}_{ak} - \omega^{a}_{k}F^{\lambda}_{ja}$

$\overset{(0)}{\nabla}_{h}F^{\lambda}_{jk}\cdot dy^{h} + \overset{(1)}{\nabla}_{h}F^{\lambda}_{jk}\delta y^{(1)h} + \overset{(2)}{\nabla}_{h}F^{\lambda}_{jk}\delta y^{(2)h}$

又、(1.5)ノ積分可能條件ハ次ノ通リデアル。

(2,2)

a) $$\overset{(0)(0)\lambda}{S}_{\mu\sigma(0)}\xi^{\mu}_{j}\xi^{\sigma}_{k}+\frac{1}{3}\left(\overset{(0)(1)\lambda}{S}_{\mu\sigma(0)}\xi^{\mu}_{j}\mathcal{J}^{\sigma}_{(2)k}-\overset{(0)(1)\lambda}{S}_{\mu\sigma(0)}\xi^{\mu}_{k}\mathcal{J}^{\sigma}_{(2)j}\right)$$

$$=\xi^{\lambda}_{i}\overset{(0)(0)i}{S}_{jk(0)}+F^{\lambda}_{jk}-F^{\lambda}_{kj}$$

b) $$\overset{(0)(1)\lambda}{S}_{\mu\sigma(0)}\xi^{\mu}_{j}\xi^{\sigma}_{k}=\xi^{\lambda}_{i}\overset{(0)(1)i}{S}_{jk(0)}+\frac{2}{3}\mathcal{J}^{\lambda}_{(2)j(2)k}$$

c) $$\overset{(0)(0)\lambda}{S}_{\mu\sigma(1)}\xi^{\mu}_{j}\xi^{\sigma}_{k}+\frac{1}{3}\overset{(0)(1)\lambda}{S}_{\mu\sigma(1)}\xi^{\mu}_{j}\mathcal{J}^{\sigma}_{(2)k}$$

$$-\frac{1}{3}\overset{(0)(1)\lambda}{S}_{\mu\sigma(1)}\xi^{\mu}_{k}\mathcal{J}^{\sigma}_{(2)j}-\frac{1}{3}\overset{(0)(2)\lambda}{S}_{\mu\sigma(1)}\xi^{\mu}_{k}\overset{(1)}{\overline{\nabla}}_{j}\mathcal{J}^{\sigma}$$

$$+\frac{1}{3}\overset{(0)(2)\lambda}{S}_{\mu\sigma(1)}\xi^{\mu}_{j}\overset{(1)}{\overline{\nabla}}_{k}\mathcal{J}^{\sigma}=\xi^{\lambda}_{i}\overset{(0)(0)i}{S}_{jk(1)}$$

$$+\frac{1}{3}\mathcal{J}^{\lambda}_{(2)i}\overset{(0)(0)i}{S}_{jk(0)}+\frac{1}{3}\overset{(0)}{\nabla}_{k}\mathcal{J}^{\lambda}_{(2)j}-\frac{1}{3}\overset{(0)}{\nabla}_{j}\mathcal{J}^{\lambda}_{(2)k}$$

d) $$\overset{(0)(0)\lambda}{S}_{\mu\sigma(2)}\xi^{\mu}_{j}\xi^{\sigma}_{k}+\frac{1}{3}\overset{(0)(1)\lambda}{S}_{\mu\sigma(2)}\xi^{\mu}_{j}\mathcal{J}^{\sigma}_{(2)k}$$

$$-\frac{1}{3}\overset{(0)(1)\lambda}{S}_{\mu\sigma(2)}\xi^{\mu}_{k}\mathcal{J}^{\sigma}_{(2)j}+\frac{1}{9}\overset{(1)(1)\lambda}{S}_{\mu\sigma(2)}\mathcal{J}^{\mu}_{(2)j}\mathcal{J}^{\sigma}_{(2)k}$$

$$=\xi^{\lambda}_{i}\overset{(0)(0)i}{S}_{jk(2)}+\frac{2}{3}\mathcal{J}^{\lambda}_{(2)i}\overset{(0)(0)i}{S}_{jk(1)}+\frac{1}{3}\left(\overset{(1)}{\overline{\nabla}}_{i}\mathcal{J}^{\lambda}\right)\overset{(0)(0)i}{S}_{jk(0)}$$

$$+\frac{1}{3}\overset{(0)}{\nabla}_{k}\overset{(1)}{\overline{\nabla}}_{j}\mathcal{J}^{\lambda}-\frac{1}{3}\overset{(0)}{\nabla}_{j}\overset{(1)}{\overline{\nabla}}_{k}\mathcal{J}^{\lambda}$$

e) $S^{(0)(1)\lambda}_{\mu\sigma(2)} \xi^{\mu}_{j} \xi^{\sigma}_{k} + \frac{1}{3} S^{(1)(1)\lambda}_{\mu\sigma(2)} \mathcal{J}^{\mu}_{(2)j} \xi^{\sigma}_{k}$

$$- \frac{1}{3} S^{(1)(1)\lambda}_{\mu\sigma(2)} \mathcal{J}^{\mu}_{(2)k} \xi^{\sigma}_{j} = \xi^{\lambda}_{i} S^{(0)(1)i}_{jk(2)} - \frac{2}{3} \overset{(0)}{\nabla}_{j} \mathcal{J}^{\lambda}_{(2)k}$$

$$+ \frac{2}{3} \mathcal{J}^{\lambda}_{(2)i} S^{(0)(1)i}_{jk(2)} + \frac{1}{3} \overset{(1)}{\nabla}_{k} \overset{(1)}{\nabla}_{j} \mathcal{J}^{\lambda} + \frac{1}{3} \left(\overset{(1)}{\nabla}_{i} \mathcal{J}^{\lambda} \right) S^{(0)(1)i}_{jk(0)}$$

コヽデ (2.1),(2.2) ノ諸式ノ意味ヲ考ヘナケレバナラヌ。先ヅ(0.8)ヲ見レバ

$\delta x^{(1)\lambda} = 0,\ \delta x^{(2)\lambda} = 0$ ノ條件ノ下ニ恒ニ

$\underset{2}{\delta}\underset{1}{\delta} v^{\lambda} - \underset{1}{\delta}\underset{2}{\delta} v^{\lambda} = 0$ トナルタメニハ $\check{R}^{\lambda(0)(0)}_{\mu\sigma\pi} = 0$ ガ必要充分。

$\delta x^{(1)\lambda} = 0,\ \delta x^{(2)\lambda} = 0$ ノ條件ノ下ニ恒ニ

$\underset{2}{\delta}\underset{1}{\delta} v^{\lambda} - \underset{1}{\delta}\underset{2}{\delta} v^{\lambda} = 0$ トナルタメニハ $\check{R}^{\lambda(1)(1)}_{\mu\sigma\pi} = 0$ ガ必要充分。

デアルカラ

$$\left.\begin{array}{ll} \check{R}^{\lambda(0)(0)}_{\mu\sigma\pi} = 0 \text{ ノヤウナ } \nabla_x \text{ ヲ } & 0\text{-}flat \text{ ナ } \nabla_x \\ R^{\lambda(1)(1)}_{\mu\sigma\pi} = 0 \quad〃 & I\text{-}flat \quad〃 \end{array}\right\}$$

ト名付ケヤウ。

又(0.10)ヲ見ルニ

a) $\delta x^{(1)\sigma} = 0$ トシテ恒ニ $\underset{2}{\delta}\underset{1}{d} x^{\lambda} - \underset{1}{\delta}\underset{2}{d} x^{\lambda} = 0$ ト

ナルタメニハ $S^{(0)(0)\lambda}_{\mu\sigma(0)}=0$ ガ必要充分。

b) $\delta x^{(2)\sigma}=0$ トシテ恒ニ $\underset{2}{\delta}\underset{1}{\delta}x^{(1)\lambda}-\underset{1}{\delta}\underset{2}{\delta}x^{(1)\lambda}=0$ トナルタメニハ $S^{(0)(0)\lambda}_{\mu\sigma(1)}=0$, $S^{(0)(1)\lambda}_{\mu\sigma(1)}=0$ ガ必要充分。

c) $\delta x^{(1)\sigma}=0$ トシテ恒ニ $\underset{2}{\delta}\underset{1}{\delta}x^{(2)\lambda}-\underset{1}{\delta}\underset{2}{\delta}x^{(2)\lambda}=0$ トナルタメニハ $S^{(0)(0)\lambda}_{\mu\sigma(2)}=0$ ガ必要充分。

c) $dx^{\sigma}=0$ トシテ恒ニ $\underset{2}{\delta}\underset{1}{\delta}x^{(2)\lambda}-\underset{1}{\delta}\underset{2}{\delta}x^{(2)\lambda}=0$ トナルタメニハ $S^{(1)(1)\lambda}_{\mu\sigma(2)}=0$ ガ必要充分。

デ、$S^{(0)(1)\lambda}_{\mu\sigma(1)}=S^{(1)(1)\lambda}_{\mu\sigma(2)}=S^{(0)(0)\lambda}_{\mu\sigma(0)}$ ナルヲ以テ、3通リニ分類スレバヨイ。

$$\begin{cases} \text{0-symmetric} \cdots\cdots S^{(0)(0)\lambda}_{\mu\sigma(0)}=0 \\ \text{I-symmetric} \cdots\cdots S^{(0)(0)\lambda}_{\mu\sigma(1)}=0 \\ \text{2-symmetric} \cdots\cdots S^{(0)(0)\lambda}_{\mu\sigma(2)}=0 \end{cases}$$

尤モ $R^{\lambda(0)(0)}_{\tau\mu\sigma}x^{(1)\tau}=S^{(0)(0)\lambda}_{\mu\sigma(1)}$. $\left(S^{(0)(0)\lambda}_{\mu\sigma(0)}\right)_{(2)\tau}=\frac{1}{2}R^{\lambda(1)(1)}_{\tau\mu\sigma}$

ノ如キ関係ガアルカラ

$$\left.\begin{array}{l} \text{0-flat} \longrightarrow \text{I-symm} \\ \text{0-symm} \longrightarrow \text{I-flat} \end{array}\right\}$$

ナルコトガ分ル。ソコデ

(2, 1. a) ヨリ

(A) 0-flat ナ V_n ノ中ノ totally geodesic V_{n-1} ハ 又 0-flat

(2. 1. b) ヨリ

(B) I-flat ナ V_n ノ中ノ semi-geodesic V_{n-1} ハ 又 I-flat

(2. 2. a) ヨリ

(C) 0-symm. ナ V_n ノ中ノ semi-geodesic V_{n-1} ハ 又 0-symm

(2. 2. c) ヨリ

(D) I-symm. ナ V_n ノ中ノ totally geodesic V_{n-1} ハ 又 I-symm

ノヤウナコトガ見エル。(A)カラ(D)ガ, (C)カラ(B)ガ直接ニ出ル。

又 (2, 2, d) ヨリ

(E) 2-symm. ナ V_n ノ中ノ totally geodesic V_{n-1} ハ 又 2-symm

ナルコトガ知レル。

殘サレタ問題ハ V_{n-1} ノパスヲ、如何ニシテ決定スルカト云フコトデアルガ、コレハ別ノ機會ニ廻シテ、今ハ只コレダケニシテ置ク

終リニ自分勝手ナ、名称ヤ定義ヲ用ヒタ所ガ少クナイノデ此ノ点本部先生ニ御許シヲ乞フ次第デアル。又、矢野

健太郎先生ノ御指導ヲ感謝致シマス。

（昭和18年1月24日）